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The Effect of Spirality on the Evolution of Turbulence in the Solar Protoplanetary Cloud

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Abstract—We analyze the possible effect of hydrodynamic spirality that develops in a rotating disk on the synergetic structurization of cosmic matter and on the development of negative turbulent viscosity in cosmic matter within the framework of the problem of the reconstruction of the evolution of the protoplanetary cloud that surrounded the early Sun. We show that comparatively slow damping of turbulence in the disk can be partially due to the lack of reflective symmetry of the anisotropic field of turbulent velocities about its equatorial plane. We formulate the general concept of the development of energy-intensive coherent mesoscale vortex structures in the thermodynamically open system of turbulent chaos associated with the realization of inverse cascade of kinetic energy in mirror–nonsymmetrical disk turbulence. Because of energy release, the inverse cascade produces a hierarchical system of mass concentrations with a fractal density distribution, which ultimately initiate the mechanisms of triggered cluster formation. We use the methods of nonequilibrium thermodynamics to prove the possibility of the development of negative viscosity in the three-dimensional case in terms of the two-scale hydrodynamic description of maximally developed disk turbulence. Negative viscosity in a rotating disk system appears to be a manifestation of cascade processes in spiral turbulence where inverse energy transfer from small to larger vortices occurs. Within the framework of asymmetric mechanics of turbulized continua, we physically substantiated the phenomenological formula for the turbulent stress tensor of Wasiutynski, which is widely used in the astrophysical literature to explain the differential rotation of various cosmic objects by "anisotropic viscosity." The aim of our study is, first and foremost, to improve a number of representative hydrodynamic models of cosmic natural turbulized media, including the birth of galaxies and galaxy clusters, birth of stars from the diffuse medium of gas and dust clouds, formation of accretion disks and subsequent accumulation of planetary systems, and also the formation of gaseous envelopes of planets, atmospheres, etc. This paper continues the application of stochastic and thermodynamic approach to the synergetic description of the structured turbulence of astrogeophysical systems, which we have been developing in a series of our papers (Kolesnichenko, 2004, 2005; Kolesnichenko and Marov, 2006; Marov and Kolesnichenko, 2002, 2006).

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INTRODUCTION

Understanding the evolution of the solar protoplanetary cloud is a necessary prerequisite for solving the problem of the formation of the Earth and planets, which is deeply associated with the fundamental cosmogonic problem whose solution is now a major task of science (Schmidt, 1957; Safronov, 1982; Galimov, 2001). According to modern concepts, planets are formed after the loss of gravitational stability by the dust and gas subdisk produced as a result of differential rotation of the protoplanetary matter in an orbit around a sunlike star and accretion processes accompanying the settling of the dust component toward the equatorial plane of the disk[1] perpendicular to the rotation axis z (see, e.g., Toomre, 1964; Safronov, 1969, 1982, 1987; Goldreich and Ward, 1973; Nakagawa et al., 1986, and Youdin and Shu, 2002). It has now become clear that planets of the Solar System were formed just from the subdisk matter via the formation of individual discrete concentration centers and their subsequent growth. It is important that one of the key viewpoints in astrophysics associates the formation and evolution of circumstellar gas and dust disks of any kind with their turbulent nature (Zel'dovich, 1981; Fridman, 1989; Dubrulle, 1993; Balbus and Hawley, 1998; Richard and Zahn, 1999). In particular, the Reynolds number $\mathbf{Re}_{\mathrm{glob}}$ for the solar protoplanetary disk of radius R rotating with an angular velocity Ω was found to exceed 10^{10} (here ν is the kinematic viscosity, which we hereafter assume to be constant).

According to modern concepts, the most likely causes of turbulence generation in astrophysical disks are a large-scale shear flow of differentially rotating cosmic matter (Gor'kavyi and Fridman, 1994; Fridman

[1] Flattening of the rotating protoplanetary cloud is mostly a consequence of the opposition of the two main dynamic forces, gravitational and centrifugal. In the case of equilibrium between these two forces, weaker factors, such as thermal and viscous processes, self-gravity of the disk, and electromagnetic phenomena, become important for the evolution of the cloud.

et al., 2003) and chaotic magnetic fields (see Armitage et al., 2001), whose energy is often comparable to the energy of hydrodynamic turbulence[2]. Hence, accretion disks have substantial turbulent viscosity, which, combined with differential rotation of matter, provides a permanent internal source of thermal energy.

Thus, synergetic processes of self-organization (which ultimately result in the structurization of any astrophysical disks) in a thermodynamically open subsystem of turbulent chaos (see Kolesnichenko, 2003, 2004) against the background of averaged large-scale shear flow of cosmic matter[3] associated with its differential rotation are mechanisms of extreme importance that form the properties of the protoplanetary cloud at various stages of its evolution, including the development of a viscous accretion disk around the young Sun in the T Tauri phase, the formation of the dust and gas subdisk, its disruption as a result of gravitational instability, and the development of discrete concentration centers with the subsequent densification and growth of planetesimals, from which the planetary system was formed. This is also true for the development at the initial stage of the evolution of turbulized disk matter of various mesoscale relatively stable coherent vortical structures, which appear to provide the most favorable conditions for the mechanical and physicochemical interaction between the particles of matter (see Barge and Sommeria, 1995; Tanga et al., 1996; Nhavanis, 1999, and Kolesnichenko, 2005), resulting in the intensification of phase transitions and processes of heat and mass exchange between different regions of the multiphase disk system, spontaneous formation and growth of condensed dust clusters,[4] substantial modification of the spectrum of oscillations, etc. During later stages of the evolution of the protoplanetary cloud, as the disk cools down, solid particles condense and grow in size (mostly as a result of coagulation), and the gas dissipates from the disk system into the interstellar space, the dynamical, energetic, and optical roles of the dust component increase substantially. As a result of the growing inertia of particles, they all begin participate to a smaller degree in the pulsational (vortical) motion of the gas suspension, which ultimately results in their effective convergence to the equatorial plane ($z = 0$) of the disk. Thus, contrary to what many researchers believe, the turbulence of the disk medium somehow contributes to the formation of the dust and gas subdisk, whose gravitational instability ultimately results in the formation of planetesimals (see Kolesnichenko and Marov, 2006).

In view of the above, the problem of maintaining turbulence (unordered chaotic motions) in the protoplanetary cloud for a long time becomes very important,[5] because the intensity of turbulization of cosmic matter at different stages of the disk evolution determines to a considerable degree the possible mechanisms of the formation of planets (Safronov, 1969). Moreover, the evolution of structurized large-scale turbulence that redistributes the initial angular momentum and the cloud material (outer parts outward and inner parts toward the Sun) along the disk radius is associated with the problem of the present-day distribution of mass and angular momentum among the Sun and planets[6]. To produce the present-day distribution of these quantities, a starward mass flow must exist in the entire disk or in a substantial part of it over the entire T Tauri phase[7]. In a differentially rotating Keplerian disk (which can be used as a first-approximation model of the solar protoplanetary disk), the angular velocity of average rotation $\mathbf{\Omega}(\mathbf{r})$ increases as $|\mathbf{r}|^{-3/2}$, i.e., the rotation velocity of mass layers, increases toward the central body. It thus follows that the turbulent flow of momentum (mass) directed toward the inner disk layers is, generally speaking, a manifestation of negative turbulent viscosity,[8] because the flow transfers averaged momentum from slower rotating outer parts of the disk to its faster rotating inner parts (see Starr (1968); IX.

[2] Chaotic magnetic fields, which are dragged along with the accreting plasma, are mixed by the differential rotation of the disk, and undergo reconnection at the interfaces between chaotic cells, must also appreciably contribute to the viscosity in the inner region of the disk and in the outer layers of its atmosphere, where the matter reaches sufficiently high degree of ionization. Large-scale magnetic fields (see Eardley and Lightman, 1975) may also play an important part in the physics of accretion.

[3] Turbulence in the disk is usually viewed as an essentially stochastic phenomenon described by averaged hydrodynamic equations with the Reynolds stress tensor including the effect of the small-scale background field of velocity fluctuations.

[4] One of the possible scenarios of the formation and growth of dust particles in plasma consists of the following stages: first, the primary clusters are formed; after these clusters reach the critical size, the stage of heterogeneous condensation begins; at the next stage, processes of coagulation and agglomeration (cohesion) come to the fore; finally, at the last stage, the surface recombination of ions becomes the most important factor, which results in permanent deposition of matter onto the surfaces of isolated multicharged particles.

[5] According to early estimates of Von Weizsacker (1948), a mean turbulent velocity on the order of one-tenth of the orbital velocity results in a time of the cloud disruption on the order of 10^3 years, whereas, according to the same author, 10^8 years are required for planets to form.

[6] Recall that the Sun accounts for 99.87% of the mass and only for 2% of the angular momentum of the system. Such a disparity of the mass and angular momentum is difficult to explain while modeling the protostar collapse and the disk formation

[7] Because of the of the forces of viscous friction (which arise as a result of the relative displacement of the elements of the gaseous suspension in their orbital motion) the matter of the inner disk regions drifts toward the proto-Sun along a very low-angle spiral trajectory, whereas its angular momentum is transferred outward—from inner to outer disk regions.

[8] By some internal processes, e.g., by systematic transformation of heat into kinetic energy within the framework of individual perturbations (Starr, 1968).The averaged flow has a longer scale length of motion than the pulsation flow; therefore, negative viscosity involves energy transfer along the spectrum from longer to shorter scale lengthsNegative viscosity is a property of a statistic ensemble of chaotic vortical motions of rotating gaseous matter. This property describes the ability of the matter to transfer statistically averaged momentum from the spatial domains.

Application to the Circumsolar Nebula). Although various sources for generation of turbulence during different stages of the evolution of the circumsolar protoplanetary cloud (or its individual parts) and various mechanisms for the transfer of mass and angular momentum in disks have been suggested in the literature (see, e.g., a thorough review by Makalkin (2003) and extensive list of references therein), these results still require further confirmation and development.

It is important that studies of disk turbulence carried out by various authors are based mostly on the classical concept of its statistical homogeneity and local isotropy (see Kolmogorov, 1941, 1962). Local isotropy of small-scale turbulence implies the invariance of the pulsation velocity field **u** both with respect to rotations of the reference frame and a mirror reflection in arbitrary plane. At the same time, according to modern concepts, the turbulence pattern in free shear layers of differentially rotating disk matter has to a certain extent the form of a "double"[9] anisotropic system consisting of an ensemble of moving and interacting macro- and mesoscale spiral vortical formations superimposed on a background of small-scale pulsational velocities (turbulent chaos); note that small-scale vortical motions may be partially organized themselves. Such a phenomenon is due to the insufficiently studied tendency of turbulent flows to self-organize into various coherent structures in the case of high Reynolds numbers (see, e.g., Khlopkov et al., 2002). Moreover, in a turbulized protoplanetary cloud, like in any rotating gaseous object (with internal sources of heat), the so-called spirality density $\mathbf{u} \cdot \mathrm{rot}\, \mathbf{u}$ (a pseudoscalar, which reverses its sign in the case of mirror reflection) develops, which also leads to anisotropy of small-scale turbulence, which in this case has a gyrotropic nature (see Vainshtein et al., 1980; Krause and Radler, 1980). The latter means that, in the case of a small-scale vortical motion, left-rotating motions in the ensemble may be more likely than right-rotating or vice versa. It thus follows that many important hydrodynamic parameters of cosmic matter depend on the magnitude and direction of the vector of the angular velocity of the rotating protoplanetary cloud $\mathbf{\Omega}(\mathbf{r})$. These parameters include statistical characteristics of the field of pulsational velocity such as the mean spirality[10] H (this quantity has no reflection symmetry). We show below that the effect of the mean spirality on cascade energy processes in three-dimensional gyrotropic turbulence may explain the possible effect of negative viscosity in the disk.[11]

Note that this phenomenon is usually explained in terms of the theory of two-dimensional turbulence, which provides for inverse transport of energy from small to larger vortices (see, e.g., Monin and Yaglom, 1996). Negative viscosity in the Solar System is known to show up, in particular, in global mass circulations on the Sun, Jupiter, Saturn, Venus (and, possibly, on Uranus and Neptune), and in circulations of the terrestrial atmosphere and ocean (see Monin et al., 1989). Large-scale motions on the spherical surfaces of these cosmic bodies can be analyzed in terms of two-dimensional hydrodynamics, because the horizontal sizes of flows far exceed the local density scale height (see Starr, 1968; Sivashinsky and Frenkel, 1992; Vergassola et al., 1993; Gama et al., 1994). Such an approach can apparently also be partially implemented in the case of the disk turbulence under consideration, because shear mass motions in thin astrophysical disks with the thickness-to-radius ratio much smaller than unity ($h_{\mathrm{disk}}/R \ll 1$) must also possess certain properties of two-dimensional[12] turbulence (see, e.g., Bodenheimer, 1995; Klahr and Bodenheimer, 2003).

Unfortunately, the application of the theory of two-dimensional turbulence to the models of the evolution of a thin astrophysical disk faces a purely formal problem: one has to expect condensation of energy on a certain maximally accessible scale length, which lies between the pumping scale and the scale length of the system. In geophysical hydrodynamics, such problems can be overcome because of skin friction, orography, radiation energy cooling, etc., which result in energy sink and determine, in the long run, the quasi-equilibrium state of the turbulent field. The manifestations of such factors include, in particular, the existence of certain external scale lengths (e.g., the Rossby–Obukhov radius), which limit the propagation of energy toward increasingly larger-scale motions and thereby determine the typical sizes of observed coherent vortical formations. However, it is not so easy to avoid such problems when using two-dimensional turbulence in an evolutionary accretion-disk model (a system without clearly defined boundaries), because this would require us to introduce a virtual long-wavelength dissipation (and such an approach puts us onto a purely speculative path) or agree with the presence of rapidly decaying turbulence, which is inconsistent with the relatively long-term (up to the time of the formation of planetesimals) sustenance of the chaotic velocity field in the disk. Hence, the theory of two-dimensional turbulence is of limited interest as far as disks are concerned.

At the same time, we have already mentioned above that real turbulence in an astrophysical disk has a gyrotropic nature, because, in the case of fast rotation of the disk matter and inhomogeneous distribution of the

[9] Townsend (1976) called coherent vortical formations and small-scale turbulence a manifestation of the "double" structure of turbulence, thereby emphasizing the effect of organized vortical motions on the processes of turbulent transport in shear layers.

[10] The conservation of the mean spirality in nonviscous liquid flows was discovered not very long ago by Moreau (1961).

[11] This phenomenon is the hydrodynamic analogue of the alpha effect in magnetic hydrodynamics (Steenbeck et al., 1966), which explains the growth of the large-scale magnetic field (dynamo effect) in the case of turbulent motion of a conducting continuum with the breakdown of invariance with respect to the change of parity).

[12] It would be more correct to speak about quasi-two-dimensional turbulence, where motions are approximately two-dimensional; i.e., they can be described by two-dimensional hydrodynamic equations, but with additional special terms.

intensity of turbulent pulsations, the field of pulsational velocities **u**, in the general case, is reflection-symmetric with respect to the transformation $z \longrightarrow -z$. Moffatt (1969) was the first to point out the importance of the spirality of localized vortex perturbations for three-dimensional hydrodynamics of turbulized liquid. He was also the first to discover the integral invariant $H \equiv \langle(\mathbf{u} \cdot \text{rot }\mathbf{u})/2\rangle$ associated with this spirality, which characterizes the degree of connectedness of vortical formations in the flow (see, e.g., Saffman, 1992; Alekseenko et al., 2005) and remains constant along the trajectory of motion of any liquid particle of a nonviscous medium.[13] The existence of this additional nonviscous invariant for three-dimensional turbulence immediately implies a certain degree of freedom for the energy cascade process, because the two quantities (the average energy $E \equiv \langle|\mathbf{u}|^2/2\rangle$ and the average spirality H), which are conserved in nonlinear interactions in the inertial interval of the energy spectrum, participate simultaneously in the cascade turbulent process. By analogy with two-dimensional flows of an incompressible nonviscous fluid (which also involves cascade transfer of two velocity-quadratic integrals—the integrals of energy, E, and enstrophy $\Omega \equiv \langle(\text{rot }\mathbf{u})^2/2\rangle$), in principle, a mode of turbulent motion is possible that involves a cascade of these conserved quantities moving toward opposite ends of the spectrum where the direct cascade of spirality toward shorter scales should be accompanied by a reverse cascade of energy toward longer scales.

It follows from the above that an adequate mathematical model of the evolution of the real anisotropic field of velocity pulsations in a protoplanetary cloud is apparently difficult to develop without the allowance for the symmetry laws of rotating turbulence. The effect of fluctuations of background spirality on the development of negative viscosity in the disk and on the structurization of differentially rotating disk matter has not yet been discussed in detail in the literature, and therefore we attempted to fill this gap in this paper.

TWO-LEVEL DESCRIPTION OF TURBULENCE IN THE DISK

Hereafter, we proceed from the concept of the two-level[14] macroscopic description of the turbulized medium of a protoplanetary cloud in terms of two interacting continua (mutually open subsystems), which simultaneously and continuously fill the same volume of coordinate space—the subsystems of averaged motion and of turbulent chaos.[15] The continuum of averaged motion, obtained by theoretic-probabilistic averaging of instantaneous hydrodynamic equations, serves to study the evolution of averaged hydrodynamic fields, including large vortical formations in the disk. The subsystem of turbulent chaos (in the general case, the vortical anisotropic continuum with an internal structure [16] is actually the turbulent velocity field $\mathbf{u}(\mathbf{r}, t)$ associated with stochastic small-scale pulsational motion of whirling[17] liquid (for which $\omega \equiv \text{rot}\,\mathbf{u} \neq 0$). Such a division of the real turbulent flow into the imaginary averaged and pulsational flows depends, generally speaking, on the choice of the spatiotemporal averaging domain (provided that ergodicity conditions are satisfied), for which the mean values of local hydrodynamic variables (which are continuous functions of coordinates $\mathbf{r} = (x, y, z)$ and time t) are known; hence, this division is arbitrary to a certain degree.

To compose the full set of equations of motion for turbulized liquid, which is characterized by two linear scale lengths of motion—L (external) and l_0 (internal)—it is convenient to introduce two coordinate systems—the microscale coordinate system $x'_j\,(\delta x'_j \sim l)$ and the macroscale coordinate system $x_j(dx_j \sim \Lambda \gg l)$. These coordinate systems subdivide space into elementary volumes[18] $\delta\mathbf{r}'$ and $d\mathbf{r}'$, respectively. Hereafter, we assume that $L \gg \Lambda \geq l_0$ and $l_0 \gg l \gg l_\nu$. The quantity L is the integral turbulence scale (the scale length of motion of the system); l_0 (the size of a turbulent "mole") is equal to the scale length of the internal motion or the state of the medium, and l_ν is the molecular microscale, which is almost equal to zero. One can formulate the problem of deriving hydrodynamic equations of motion on macroscale x_j from the known Navier–Stokes equations on microscale x'_j. The associated problem of averaging is one of the central problems in continuum mechanics, and, in the case of such a complex system as a turbulized liquid, the method of averaging often determines the very construction of the macroscopic

[13]Recall that, according to Kelvin's theorem, vortex lines are frozen into liquid if $\nu = 0$, $p = p(\rho)$ and external forces per unit mass are conservative, implying that knots and linkages of vortex lines are inevitably conserved (Alekseenko et al., 2005).

[14]This approach is essentially an application of multiscale methods to turbulence (see, e.g., Dubrulle and Frisch, 1991; Fannjiang and Papanicolaou, 1994).

[15]This very description of developed hydrodynamic turbulence served as the starting point that allowed us to begin the development of models of structurized mesoscale turbulence as a process of self-organization in open nonequilibrium fluctuating media (see Kolesnichenko, 2002, 2003, 2005; Marov and Kolesnichenko, 2006).

[16]Kolesnichenko (2005) showed that, in the process of temporal evolution of the quasi-equilibrium subsystem of turbulent chaos, mesoscale coherent structures can be generated because of the mutual phase synchronization (coherence) of a certain ensemble of small-scale oscillatory modes with close frequencies.

[17]Vorticity plays the crucial role in the mechanics of turbulence, because it makes possible the cascade process of generation of smaller vortices by larger ones.

[18]It is important to bear in mind that here we do not speak about absolute dimensions. For example, in a hydrodynamic model of a disk medium, a physically infinitely small volume may be much greater than the volume of an entire planet.

model.[19] Hereafter, we assume that the hydrodynamic scale length of the averaged motion Λ (e.g., the mesh size of the difference grid) is such that the subsystem of turbulent chaos in the averaging domain $d\mathbf{r} \sim \Lambda^3$ contains the entire ensemble of coherent mesoscale structures whose sizes are smaller than the size of the averaging domain.[20] In this case, the effect of turbulent chaos on the averaged motion manifests itself as an additional turbulent transfer of momentum and energy by small-scale vortical formations, which requires the construction of semiempiric closure models (defining relations) for the averaged hydrodynamic equations and modeling of the effective coefficients of turbulent exchange, which allow, in particular, for the anisotropy of chaos associated with the presence of coherent mesoscale structures (Kolesnichenko, 2002).

Equations of turbulent chaos in the presence of average flow. Let the random component $\mathbf{u}(\mathbf{r}, t)$ of the velocity field $\mathbf{U}(\mathbf{r}, t)$ have a scale length l_0[21], which is small compared to the scale of hydrodynamic averaging Λ. Hereafter, we assume that the averaged velocity $\langle \mathbf{U}(\mathbf{r}, t)\rangle$ only slightly varies on any intermediate scale length l satisfying the inequality $l_\nu \ll l \ll l_0 \ll \Lambda$, and hence the methods of the theory of homogeneous turbulence (see Monin and Yaglom, 1996) can be applied on such intermediate scales. In this paper, we also assume that the disk medium is not subject to electromagnetic forces, the disk rotates about the z-axis with the Keplerian angular velocity $\Omega(r)$, and the origin of the coordinate system coincides with the center of mass of the system. We further assume, for the sake of simplicity, that the large-scale (averaged) flow reduces to differential rotation exclusively. The components of the vector of the averaged velocity $\langle \mathbf{U}\rangle$ then have the following form in cylindrical coordinates $\langle U\rangle_r = 0$, $\langle U\rangle_\varphi = \Omega(r)r$, $\langle U\rangle_z = 0$.

We thus proceed in our analysis of three-dimensional disk turbulence from the following set of instantaneous equations of motion for incompressible[22] homogeneous liquid, which includes the Navier–Stokes equation, the continuity equation, and the equation of state:

$$\begin{aligned} \partial_t \mathbf{U} + (\mathbf{U}\cdot\nabla)\mathbf{U} &= -\rho^{-1}\nabla P + \nu\nabla^2\mathbf{U} + \mathbf{g}, \\ \operatorname{div}\mathbf{U} &= 0, \quad p = p(\rho). \end{aligned} \tag{1}$$

Here, $P(\mathbf{r}, t)$ is the true (instantaneous) pressure of the disk matter; $\mathbf{g}(\mathbf{r}, t) = -\nabla\Psi$ is the vector of acceleration due to the external mass force (gravity), and $\Psi(\mathbf{r}, t)$ T) is the Newtonian gravitational potential. If the mass of the protoplanetary cloud is equal to several percent of the mass of the central body (or, more precisely, if $\mathcal{M}_{\text{disk}}/\mathcal{M}_\odot \le h_{\text{disk}}/R$, where h_{disk} and R are the disk half thickness and radius, respectively), one can neglect the self-gravity of disk particles[23]; in this case, we have $\Psi = G\mathcal{M}_\odot/|\mathbf{r}|$, $\mathbf{g} = -\nabla\Psi = G\mathcal{M}_\odot\mathbf{r}/|\mathbf{r}|^3$ (where $\mathcal{M}_\odot$ is the mass of the star, G is the gravitational constant, and $|\mathbf{r}|$ is the central radius-vector).

We now average[24] Eqs. (1) over an ensemble of identical hydrodynamic systems to derive the Reynolds equation:

$$\begin{aligned} \partial_t\langle\mathbf{U}\rangle + (\langle\mathbf{U}\rangle\cdot\nabla)\langle\mathbf{U}\rangle &= -\rho^{-1}\nabla\cdot(\mathbf{I}\langle P\rangle - \mathbf{R}) \\ &\quad + \nu\nabla^2\langle\mathbf{U}\rangle - \nabla\langle\Psi\rangle, \\ \operatorname{div}\langle\mathbf{U}\rangle &= 0, \end{aligned} \tag{2}$$

where $\mathbf{R}(\mathbf{r}, t) = -\rho\langle\mathbf{uu}\rangle$ is the Reynolds stress tensor and $\mathbf{I}$ is the unit vector.

The equations for pulsational velocity $\mathbf{u}$ can be derived by subtracting the corresponding Eqs. (2) from Eqs. (1). If we further restrict our analysis to the so-called second-order correlation approximation (see Krause and Radler, 1980), when the terms that are quadratic in velocity fluctuations can be neglected, and use the above assumption that the averaged velocity $\langle\mathbf{U}(\mathbf{r}, t)\rangle$ remains unchanged on any intermediate microscale l inside the inertial interval $l_\nu \ll l \ll l_0$, then we have to consider the following equations:

$$\begin{aligned} \partial_t\mathbf{u} + ((\langle\mathbf{U}\rangle + \mathbf{u})\cdot\nabla)\mathbf{u} &= -\rho^{-1}\nabla p + \nu\nabla^2\mathbf{u} - \nabla\psi, \\ \operatorname{div}\mathbf{u} &= 0. \end{aligned} \tag{3}$$

Which, in the case of the elementary volume $d\mathbf{r}'$ moving at the average velocity $\langle\mathbf{U}(\mathbf{r}, t)\rangle$ of the turbulized flow, acquire (in the case of the appropriate redetermi-

[19] The most clear description of modern methods of spatial averaging for turbulized liquid, which correspond to the transition from equations of motion of small elements of the continuum to the description of the same motions on macroscales, can be found in the paper by Nikolaevskiy (2003).

[20] According to available estimates, for the averaged flow to contain the bulk (80% or 90%) of the total energy of the turbulent motion, the averaging scale length Λ must be ten to twenty times shorter than the integral scale length L.

[21] In the case of small-scale turbulence, the scale length l_0 may coincide with the size of energy-containing vortices.

[22] Hereafter, for the sake of simplicity, we restrict our analysis to incompressible fluids $\langle\rho\rangle = \rho = \text{const}$. Analysis of turbulence in a compressible medium would require more mathematical efforts.

[23] In the cases where self-gravity effects are important, $\Psi = G\mathcal{M}_\odot/|\mathbf{r}| + \Psi_{\text{cr}}$, and the potential of self-gravity Ψ_{cr} satisfies the Poisson equation.

[24] Angular braces in this paper mean averaging $\langle\mathcal{A}(\mathbf{r}, t)\rangle \equiv \frac{1}{l^3}\int_{|\xi|<l}\mathcal{A}(\mathbf{r}+\boldsymbol{\xi}, t)d^2\boldsymbol{\xi}$ of an l-periodic parameter $\mathcal{A}(\mathbf{r}, t)$ over some cubic spatial domain with the edge l (such that periodic boundary conditions are satisfied on the faces of this cube) under the assumption that this averaging does not depend on the exact value of the scale; from a purely mathematical viewpoint, this definition can be considered to be identical to averaging over an ensemble of identical hydrodynamical systems in the asymptotic limit $l/\Lambda \longrightarrow \infty$. Hereafter, we use the following identity relations for periodic functions that can be proved by integration by parts: $\langle\partial_i\mathcal{A}\rangle = 0$; $\langle(\partial_i\mathcal{A})\mathcal{B}\rangle = -\langle\mathcal{B}\partial_i\mathcal{A}\rangle$; $\langle(\nabla^2\mathcal{A})\mathcal{B}\rangle = -\langle(\partial_i\mathcal{B})(\partial_i\mathcal{A})\rangle$; $\langle\mathbf{u}\cdot(\operatorname{rot}\mathbf{v})\rangle = \langle(\operatorname{rot}\mathbf{u})\cdot\mathbf{v})\rangle$; $\langle\mathbf{u}\cdot\nabla^2\mathbf{v}\rangle = -\langle(\operatorname{rot}\mathbf{u})\cdot(\operatorname{rot}\mathbf{v})\rangle$, if $\operatorname{div}\mathbf{v} = 0$.

nation of the velocity $\mathbf{u}$ in this coordinate system) the following form[25]:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{u} \equiv \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla\left(\frac{p}{\rho} + \psi\right) + \nu \nabla^2 \mathbf{u}, \qquad \operatorname{div}\mathbf{u} = 0. \tag{4}$$

Here, $p \equiv P - \langle P \rangle$ is the pressure pulsation; $\psi \equiv \Psi_{\rm cr} - \langle \Psi_{\rm cr} \rangle$ (hereafter, we assume that ψ excites $\mathbf{u}$ in some way that we do not specify here). It goes without saying that Eqs. (4) must be supplemented by periodic boundary conditions in spatial variable: $\mathbf{u}(x + nl, y + ml, z + ql) = \mathbf{u}(x, y, z)$ for all x, y, and z and all integer n, m, and q.

Laws of conservation in locally isotropic turbulence. Consider now integrated conservation laws associated with the homogeneity, isotropy, and mirror symmetry of the turbulent field $\mathbf{u}$. The identity relations given in footnote 24 can be used to derive from Eqs. (4), by integrating them over the spatial periodicity cell (a spatial cubic domain with the edge l such that periodic boundary conditions are satisfied on its faces), the following conservation laws for the averaged kinetic energy $E = \langle |\mathbf{u}|^2/2 \rangle$, enstrophy,[26] $\Omega = \langle |\boldsymbol{\omega}|^2/2 \rangle$ and the total spirality $H = \langle \mathbf{u} \cdot \boldsymbol{\omega}/2 \rangle$ (see, e.g., Frisch (1995)):

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\varepsilon = -2\nu\Omega, \quad \frac{\mathrm{d}\Omega}{\mathrm{d}t} = -\varepsilon_\Omega, \quad \frac{\mathrm{d}H}{\mathrm{d}t} = -\varepsilon_H, \tag{5}$$

where

$$\varepsilon \equiv \frac{\nu}{2}\left\langle \sum_{i,j} (\partial_i u_j + \partial_j u_i)^2 \right\rangle, \quad \varepsilon_\Omega \equiv \nu \langle |\mathrm{rot}\boldsymbol{\omega}|^2 \rangle, \qquad \varepsilon_H \equiv \nu \langle \boldsymbol{\omega} \cdot \mathrm{rot}\boldsymbol{\omega} \rangle. \tag{6}$$

Here, the quantities ε, ε_Ω, and ε_H denote the dissipation rates of averaged kinetic energy, enstrophy, and spirality per unit mass, respectively. It is evident from Eqs. (5) that the quantities E, Ω, and H remain constant in the nonviscous limit $\nu \longrightarrow 0$ 0 (and, in particular, throughout the entire inertial interval) if no dissipation and motion pumping are present.

ENERGY CASCADE IN ISOTROPIC TURBULENCE WITH MIRROR SYMMETRY

Before analyzing the possible effect of spirality on the dynamics of disk turbulence, we must recall some of the concepts and quantitative spectral characteristics of small-scale turbulence (see Batcherlor, 1953; Monin and Yaglom, 1996).

The dynamics of vorticity and the energy cascade. As we have already pointed out above, most of the studies of turbulent motion within a disk, carried out within the framework of the problem considered here, were based mostly on the concept of Kolmogorov (1941, 1962). According to this concept, in the limit of large Reynolds numbers (which correspond to large-scale motions in the flow of cosmic matter), the random nature of vortex fragmentation and the chaotic nature of the transfer of their energy downscale the cascade result in almost locally isotropic[27] stochastic regime of turbulent fluctuations within the boundaries of the spatiotemporal domain of averaging of instantaneous hydrodynamic equations despite the anisotropy, inhomogeneity, and nonstationarity of the averaged flow. In this case, the energy structure of the three-dimensional small-scale field of velocity pulsations is statistically similar for large Reynolds numbers $\mathbf{Re} \equiv u_0 l_0/\nu$ (where $u_0 = \sqrt{\langle |\mathbf{u}|^2 \rangle}$ is the characteristic velocity of pulsational velocity field and l_0 is the characteristic scale length of energy-bearing vortices), and the inertial interval of wavenumbers $k_0 \ll k \ll k_\nu$ that separates the domains of dissipation and generation of turbulent energy $E = u_0^2/2$ in the space of wavenumbers k is the wider the higher is the Reynolds number $\mathbf{Re}$. The dissipation rate of the averaged kinetic energy per unit mass ε, which is given by formula (6) in the initial Kolmogorov theory (K41), is considered to be a universal constant for the turbulent motion studied. The quantity ω also characterizes the flow of kinetic energy, which is transferred in a cascade mode without any loss along successively increasing wavenumbers $k_n \gg k_{n-1}$ ($n = 1, 2, \ldots$) (decreasing scale lengths, $l_n = 1/k_n$) within the inertial interval until the flow reaches the dissipation scale length $l_\nu = 1/k_\nu \sim (\nu^3/\varepsilon)^{1/4}$, for which the energy dissipation rate due to kinematic viscosity is equal to ε.

Under the assumption of homogeneity and stationarity of the field of pulsation velocities $\mathbf{u}(\mathbf{r}, t)$, the most important turbulence characteristics are the (two-point and two-time) correlation[28] tensor of the velocity field $\tilde{R}_{ij}(\mathbf{r}, \mathbf{x}, t, \tau) \equiv \langle u_i(\mathbf{r}, t) u_j(\mathbf{r} + \mathbf{x}, t + \tau) \rangle$ and the spectral energy tensor (see Batchelor, 1953)

$$\Phi_{ij}(\mathbf{k}, \omega) = \frac{1}{(2\pi)^4} \iint \tilde{R}_{ij}(\mathbf{x}, \tau) \exp[-i(\mathbf{k} \cdot \mathbf{x} - \omega\tau)] \mathrm{d}\mathbf{x}\mathrm{d}\tau, \tag{7}$$

which is in fact the Fourier image of the correlation tensor $\tilde{R}_{ij}$. Note that the complex tensor $\Phi_{ij}(\mathbf{k}, \omega)$ for

[25]This means that the subsystem of turbulent chaos has zero hydrodynamic velocity relative to the subsystem of averaged motion.

[26]Note that the balance equation (5) for enstrophy is valid only in the two-dimensional case.

[27]Recall that in locally isotropic turbulence, any average quantity that characterizes it is invariant with respect to any parallel translations, rotations, and mirror reflections.

[28]Note that components of the turbulent Reynolds stress tensor can be written in the form $R_{ij}(\mathbf{r}, t) = -\rho \tilde{R}_{ij}(\mathbf{r}, 0, t, 0)$,, and the total spirality, in the form $H = \varepsilon_{ijk}(\partial \tilde{R}_{ik}/\partial x_j)|_{x=0,\tau=0}$.

incompressible fluid (div $\mathbf{u} = 0$) has the following properties (which we use below)[29]:

$$k_j \Phi_{ij}(\mathbf{k}, \omega) = k_i \Phi_{ij}(\mathbf{k}, \omega) = 0. \quad (8)$$

The energy spectral function (density) $E(k, \omega)$, which is the most important quantity in the problem of homogeneous turbulence, is determined by the following integral:

$$E(k, \omega) = \frac{1}{2} \int_{S_k} \Phi_{ii}(\mathbf{k}, \omega) \mathrm{d}S, \quad (9)$$

where integration is performed in the $\mathbf{k}$-space over the sphere S_k of radius $k \equiv |\mathbf{k}|$. The total kinetic energy per unit mass transferred along the cascade can then be written in the following form[30]

$$E \equiv \frac{1}{2}\langle |\mathbf{u}|^2 \rangle = \frac{1}{2}\tilde{R}_{ii}(0, 0) = \frac{1}{2}\iint \Phi_{ii}(\mathbf{k}, \omega) \mathrm{d}\mathbf{k}\mathrm{d}\omega = \iint E(k, \omega) \mathrm{d}k\mathrm{d}\omega, \quad (10)$$

and $\Phi_{ii} \geq 0$ for almost all $\mathbf{k}$ and ω, because these diagonal elements represent the density of the kinetic energy components in the wave space.

In the wavenumber interval $k_0 \ll k \ll k_\nu$, the spectral energy tensor $\Phi_{ij}(\mathbf{k}, \omega)$, which is a second-rank isotropic tensor, is statistically unrelated to the source of energy, which is restricted by the wavenumber k_0, and therefore we can assume that it must be determined by the energy dissipation rate ε exclusively:

$$\Phi_{ij}(\mathbf{k}, \omega) = \frac{E(k, \omega)}{4\pi k^4}(k^2 \delta_{ij} - k_i k_j). \quad (11)$$

As follows from dimensional considerations, the spectral energy density must be described by the Kolmogorov formula

$$E(k) = C\varepsilon^{2/3} k^{-5/3} \ (k_0 \ll k \ll k_\nu), \quad (12)$$

where C is a dimensionless constant on the order of unity.

Two-dimensional turbulence. In the case of developed two-dimensional turbulence[31] in a noncompressible fluid, Kolmogorov-type theories have two positive definite quantities that are conserved in the nonviscous limit. They are the averaged kinetic energy $E = \langle |\mathbf{u}|^2/2 \rangle$ and the enstrophy $\Omega \equiv \langle |\mathrm{rot}\,\mathbf{u}|^2/2 \rangle$, which, if generated in a uniform stream on some intermediate scale lengths of energy pumping k_l, which are far from the dissipation scale k_ν, both become involved in the cascade process. As is evident from formula (5), in the case of finite viscosity in a two-dimensional flow, the enstrophy can only monotonically increase with time along with the quantity $\varepsilon = 2\nu\Omega$. This is due to the fact that the mechanism of stretching of vortex tubes, which ensures the growth of enstrophy in three-dimensional flows, is blocked in the case of a two-dimensional flow.

In the general three-dimensional case, the Fourier transform of the vortex field $\boldsymbol{\omega} \equiv \mathrm{rot}\,\mathbf{u}$, whose components have the form $\omega_i = \varepsilon_{ijk}\partial_j u_k$, is evidently equal to $\hat{\boldsymbol{\omega}} \equiv i\mathbf{k} \times \hat{\mathbf{u}}$, and its spectral tensor has the form $\Omega_{ij}(\mathbf{k}, \omega) = \varepsilon_{imn}\varepsilon_{jpq}k_m k_p \Phi_{nq}(\mathbf{k}, \omega)$, where ε_{jpq} is the asymmetric Levi-Civita tensor.[32] In particular, we derive from this, in view of formula (8),

$$\Omega_{ij}(\mathbf{k}, \omega) = k^2 \Phi_{ij}(\mathbf{k}, \omega) = \frac{E(k, \omega)}{4\pi k^2}(k^2 \delta_{ij} - k_i k_j). \quad (13)$$

Note that the formula $\Omega = \varepsilon/2\nu$ implies that the spectrum of the root-mean-square velocity vortex Ω coincides with the spectrum of viscous dissipation of kinetic energy ε. The following formula is an immediate implication of Eq. (13):

$$\Omega \equiv \langle |\boldsymbol{\omega}|^2/2 \rangle = \int \Omega(k) \mathrm{d}k = \iint k^2 E(k, \omega) \mathrm{d}k\mathrm{d}\omega, \quad (14)$$

where the corresponding spectral density for the root-mean-square velocity vortex has the form

$$\Omega(k) = k^2 E(k) = C\varepsilon^{2/3} k^{1/3}. \quad (15)$$

It is shown in the theory of two-dimensional turbulence (see, e.g., Monin and Yaglom, 1996; Chapter 26) that relationship (15) between the spectral densities of energy $E(k)$ and enstrophy $\Omega(k)$ forbids the simultaneous downscale transfer of these quantities.[33] Energy in the two-dimensional case is transferred to large and not to small (as in the three-dimensional case) scale lengths, whereas the flow of enstrophy is directed toward short scales. Note that there are two inertial intervals in the case of developed two-dimensional turbulence. For small wavenumbers $k_0 < k < k_l$, the cascade process is determined by the energy dissipation rate and the dimension analysis yields classical formula (12) for the spectral density with the only essential difference from the three-dimensional case that the energy flow in the inertial interval with spectrum (12) is directed from shorter to longer scales. For large wavenumbers ($k_\nu > k > k_l$), the

[29] Hereafter, summation is to be performed over repeated indices if not otherwise stated.

[30] Hence, the quantity $E(k, \omega)\mathrm{d}k\mathrm{d}\omega$ can be interpreted as the kinetic energy (from all wavenumbers with a fixed absolute value) contained in the wavenumber interval $(k, k + \mathrm{d}k)$ and in the frequency interval $(\omega, \omega + \mathrm{d}\omega)$.

[31] Strictly two-dimensional turbulence is only a mathematical idealization and is never realized in nature.

[32] ε_{ijk} is the tensor, which is equal to $\varepsilon_{ijk} = 0$ if i, and are not all different, and $\varepsilon_{ijk} = 1$ or -1 if i, , and are all different and arranged in a cyclic or anticyclic order, respectively.

[33] Kraichnan (1967) was the first to propose the hypothesis of inverse cascade of energy in two-dimensional turbulence. The same author (Kraichnan, 1976b) interpreted inverse cascade in terms of negative turbulent viscosity. According to some estimates, inverse cascade, which has been confirmed many times by numerical simulations, is one of the most important results in the theory of developed turbulence since Kolmogorov's works (see Gama et al., 1994; Frisch, 1995).

enstrophy dissipation rate $\varepsilon_\Omega \equiv \nu\langle|\mathrm{rot}\boldsymbol{\omega}|^2\rangle$ becomes an additional determining quantity, and dimension analysis yields a different spectral distribution of kinetic energy by the absolute values of the wavenumber k:

$$E(k) = C_\Omega \varepsilon_\Omega^{2/3} k^{-3}, \quad (16)$$

which describes the inertial interval of enstrophy transfer. Note that, in the two-dimensional case, the enstrophy cascade is a direct one; i.e., enstrophy is transferred from longer to shorter scale lengths. Formula $k_\nu \sim (\varepsilon_\Omega/\nu^3)^{1/6}$ determines the boundary of the interval of the enstrophy transfer.

Thus, in the case of two-dimensional turbulence, we have a reverse cascade of kinetic energy E, where the energy of small-scale chaotic motion is used for the energy pumping of mesoscale vortex structures. This effect, which came to be known as negative viscosity (see Starr, 1968, Vergassola et al., 1993; Gama et al., 1994), is typical, as we pointed out above, for many quasi-two-dimensional cosmic objects. We now pass to the analysis of real three-dimensional hydrodynamic turbulence in natural media.

ENERGY AND SPIRALITY CASCADES IN A REFLECTION NONINVARIANT DISK TURBULENCE

The breakdown of mirror symmetry in a protoplanetary disk. Hence, the analysis of turbulence fields, which statistically satisfy certain symmetry conditions (as in the case of locally isotropic turbulence), can be seen to result in a number of important mathematical simplifications. Although mirror symmetry is a fundamental property of hydrodynamic equations, the definition of some of the important hydrodynamic parameters includes the notions of righthandedness (lefthandedness). Such quantities include, in particular, the vorticity of the field of velocity pulsations $\boldsymbol{\omega} \equiv \mathrm{rot}\mathbf{u}$. As we pointed out above, such a statistical characteristic of small-scale turbulence devoid of mirror symmetry as the total spirality (a pseudoscalar[34] of the velocity field, determined by the mean (see footnote 24))

$$H \equiv \frac{1}{2}\langle \mathbf{u}\cdot\boldsymbol{\omega}\rangle = \frac{1}{2l^3}\int\limits_{|\boldsymbol{\xi}|<l} \mathbf{u}(\mathbf{r}+\boldsymbol{\xi}, t)\cdot\boldsymbol{\omega}(\mathbf{r}+\boldsymbol{\xi}, t)\mathrm{d}^3\boldsymbol{\xi}, \quad (17)$$

may be of considerable interest for differentially rotating matter. The field of pulsation velocities $\mathbf{u}$ with nonzero average spirality has the form of anisotropic continuum[35] formed by the ensemble of arbitrarily oriented small-scale spiral vortices where, e.g., right-handed motions (vortical structures) are more likely than left-handed structures. Thus, the quantity H, related to the topological structure of a complex vorticity field, is a fundamental measure of the "lack of reflection symmetry" in a turbulent flow.

According to Moffatt (1981), the spirality of turbulent field, "which is a quadratic invariant of some localized motion of a fluid (under the conditions determined above), has a status comparable to the status of the perturbation kinetic energy."[36] Note that only the analysis of the background anisotropic turbulence with a magnetic field and devoid of mirror symmetry (whose simplest measure is provided, in particular, by the hydrodynamic[37] spirality H) allowed a breakthrough to happen in the understanding of turbulent magnetic dynamo in astrophysics (the so-called α -effect) (see, e.g., Vainshtein et al., 1980; Parker, 1979; Krause and Radler, 1980). At the same time, as we show below, the quantity H can be also considered as a statistical characteristic of an anisotropic field of pulsation velocities, which is capable of developing the effect of negative viscosity in a rotating medium (in particular, in a protoplanetary disk).

It is known that the spirality of the velocity field can be efficiently generated in a mirror asymmetric (i.e., not invariant with respect to parity reversal) field of small-scale random velocities $\mathbf{u}$, e.g., in turbulence that rapidly rotates about a fixed axis (see, e.g., Steenbeck et al., 1966). In particular, spirality may naturally develop in a protoplanetary cloud because of its rotation and nonuninform distribution of density (density stratification) or intensity of turbulent pulsations in zones with developed convection. Here we give some preliminary arguments in favor of the hypothesis that, at some heliocentric distances, vertical thermal turbulent convection in the disk (during certain stages of the disk evolution) is very likely to result in left-handed spiral motions in the regions between its equatorial plane and the "upper" surface. Indeed, the ascending matter should expand and rotate under the action of the Coriolis forces and this process results in lefthanded spiral motion. The descending matter should compress and has to rotate in the opposite direction under the

[34]Recall that vectors A, which behave as $\mathbf{A}^{\mathrm{ref}}(\mathbf{r}, t) = -\mathbf{A}(-\mathbf{r}, t)$, are referred to as polar vectors, whereas vectors satisfying the relation $\mathbf{A}^{\mathrm{ref}}(\mathbf{r}, t) = \mathbf{A}(-\mathbf{r}, t)$ are referred to as axial vectors or pseudovectors (the superscript "ref" hereafter means the operator of reflection in an arbitrary plane or in an arbitrary point). The scalar $V^{\mathrm{ref}} \equiv (\mathbf{A}^{\mathrm{ref}} \times \mathbf{B}^{\mathrm{ref}}) \cdot \mathbf{C}^{\mathrm{ref}} = -(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = -V$, which depends on the use of righthandedness, is a pseudoscalar; the last statement means that this scalar reverses its sign if we change over from a righthanded coordinate system to a lefthanded one.

[35]Recall also that, in his studies of small-scale properties of flows (the random-cascade model), Kolmogorov (1941, 1962) did not take into account any spatial structures that could possibly exist in the turbulent flow. However, as we pointed out above, according to modern concepts, vortical structures (threads) are almost always present in a turbulent flow on small scales and these structures may affect the flow properties in the inertial interval as well.

[36]The question so far remains open about other possible integral invariants (see Edwards, 1967, 1968) besides the classical invariants of energy, momentum, and angular momentum, which characterize to a certain degree the preserved topological configuration of vortical threads.

[37]The quantity H is often referred to as "hydrodynamic" spirality in order to distinguish it from magnetic spirality.

action of Coriolis forces and its motion is again lefthanded. It is evident that right-handed spiral motions should dominate in the "lower" part of the disk. The balance of left- and right-handed spiral motions may be established in the vicinity of the equatorial plane of the disk only in the absence of gradients of turbulence intensity, i.e., at the very late stages of the evolution of a differentially rotating protoplanetary cloud.

Effect of spirality on the energy cascade. Thus, we can imagine a situation where the source of kinetic energy (e.g., as a result of the interaction of Archimedes and Coriolis forces (see below) in the differentially rotating disk matter) at wavenumbers k_0 generates a nonzero total spirality H. We have shown above that the total spirality of the pulsational velocity field in the inertial interval is a conserved quantity (like the total energy), $\mathrm{d}\langle \mathbf{u} \cdot \boldsymbol{\omega}\rangle/\mathrm{d}t = 0$. Note that the generation of spirality is associated with the development of large-scale engagements[38] of vortex flow lines (see, e.g., Alekseenko et al. (2005)), which are conserved in a cascade (not nonviscous) process in the inertial domain, but are destroyed by viscosity on scale lengths $l_\nu = k_\nu^{-1}$.

By analogy with the definition of quantity $E(k)$ (9), we can define the spectral spirality density

$$F(k, \tau) \equiv i\int_{S_k} \varepsilon_{jkl} k_k \Phi_{jl}(\mathbf{k}, \tau)\mathrm{d}S, \tag{18}$$

where integration is performed in the $\mathbf{k}$ space over a sphere S_k of radius $k \equiv |\mathbf{k}|$. We thus have for the total spirality H

$$H = i\varepsilon_{jkl}\iint k_k \Phi_{jl}(\mathbf{k}, \tau)\mathrm{d}\mathbf{k}\mathrm{d}\tau = \iint F(k, \tau)\mathrm{d}k\mathrm{d}\tau. \tag{19}$$

The function $F(k, \tau)$ is, in view of relation (18), real and it is a pseudoscalar; it is equal to zero if the field of pulsational velocities $\mathbf{u}$ is statistically invariant with respect to the reflection transform (e.g., a parity transform of the form $x' = x$, $y' = y$, $z' = -z$, which describes a mirror reflection in the $z = 0$ plane).

In the case of isotropic and reflection-nonsymmetric turbulence, the functions $E(k, \tau)$ and $F(k, \tau)$ are sufficient for the complete determination of the spectral tensor $\Phi_{il}(\mathbf{k}, \tau)$. In this case, the most general formula for the uniform field $\mathbf{r}$ (which is stationary in t) of small-scale turbulence that satisfies equalities (8), (9), and (18) has the following form (see Moffatt, 1978):

$$\Phi_{ij}(\mathbf{k}, \tau) = \frac{E(k, \tau)}{4\pi k^4}(k^2\delta_{ij} - k_i k_j) + \frac{iF(k, \tau)}{8\pi k^4}\varepsilon_{ijk}k_k. \tag{20}$$

Unlike the spectral density of kinetic energy $E(k, \tau)$, the spectral spirality density $F(k, \tau)$ may be both positive and negative. And the ambiguous role of spirality in three-dimensional cascade processes is due to this very feature, because simple arguments that imply the existence of two inertial intervals (as in the case of two-dimensional turbulence) do not work in this case. In the two-dimensional case, the spectral densities of energy and enstrophy are related by formula (15), whereas only an upper limit (the so-called "realizability" condition) can be obtained for the function $F(k)$:

$$|F(k)| \le 2kE(k). \tag{21}$$

This constraint follows, e.g., from the Cauchy–Bunyakovsky–Schwartz inequality written in the form

$$\left|\int_{S_k} \langle \hat{\mathbf{u}} \cdot \hat{\boldsymbol{\omega}}^* + \hat{\mathbf{u}}^* \cdot \hat{\boldsymbol{\omega}}\rangle \mathrm{d}S\right|^2 \le 4\int_{S_k} \langle |\hat{\mathbf{u}}|^2\rangle \mathrm{d}S \int_{S_k} \langle |\hat{\boldsymbol{\omega}}|^2\rangle \mathrm{d}S,$$

and from equalities (10), (13), and (19). Here asterisk denotes complex conjugation and the "^" sign above a symbol denotes Fourier transform.

Generally speaking, inequality (21) allows the realization of two scenarios of the behavior of spirality in a turbulent flow (Brissaund et al., 1973). First, in some cases, by analogy with two-dimensional turbulence, there is a cascade of conserved quantities toward the opposite ends of the inertial interval of the spectrum, and the direct downscale cascade of spirality H is accompanied by the synchronous inverse cascade of energy E toward larger scales. Second, there is the possibility of the simultaneous direct cascade of both quantities toward small scales. Which of the two processes occurs at the given time instant, depends on the integral properties of the system considered and also on the boundary and initial conditions.

In the first case, we use the hypothesis that the energy spectrum $E(k)$ may depend only on the wavenumber k and the spirality dissipation rate ε_H; dimension considerations yield a spectral law of the form

$$E(k) \sim \varepsilon_H^{2/3} k^{-7/3}. \tag{22}$$

The spectral spirality function $F(k)$ is determined by the process of spirality transfer from the source acting at wavenumbers k_0 to the viscous sink on wavenumbers k_ν and further. During the spirality generation, large-scale engagements of vortex lines of the flow under consideration develop, which survive in the cascade process in the inertial domain, but are destroyed by viscosity on scale lengths $l_\nu = k_\nu^{-1}$.

[38] As we already pointed out above, spirality characterizes the degree of connectedness of vortex lines in a flow. The number of turns n of a thread around another thread is characterized by the engagement $H = \pm 2n\Gamma_1\Gamma_2$, where Γ_k is the intensity of the thread and the "sign" refers to the right or left engagement, respectively. If a single vortex tube twines itself before closing, it has got a knot on it; the conservation of spirality also means that the knot structure of the vortex field is conserved. It has been also shown that the spirality invariant H is associated with a more general topological property—the so-called Hopf (1984) invariant (see Moffatt, 1984).

The second scenario assumes passive behavior of spirality in the turbulent flow. This means the realization of common Kolmogorov energy cascade $E(k)$ toward small scales with law (12). Let the spirality generation rate at wavenumbers $\sim k_0$ be equal to ε_H (see formula (6)). Spirality is generated simultaneously with energy, and therefore it is evidently constrained by an inequality of the form $|\varepsilon_H| \le k_0\varepsilon$ (Brissaund et al., 1973). If spirality is injected at the maximum rate, then

$$|\varepsilon_H| \sim k_0\varepsilon \sim u_0^3/l_0^2. \tag{23}$$

The spirality spectrum $F(k)$ must be proportional to ε_H (because of the pseudoscalar nature of both quantities), and the only possible additional parameters that may determine $F(k)$ in the inertial domain $k_0 \ll k \ll k_\nu$ are ε and k. It thus follows from dimension considerations that

$$F(k) = C_H\varepsilon_H\varepsilon^{-1/3}k^{-5/3} \; (k_0 \ll k \ll k_\nu), \tag{24}$$

where C_H is the universal constant similar to the Kolmogorov constant C. Note that equalities (23), (12), and (24) also imply inequality $|F(k)| \le 2kE(k)$, in full agreement with formula (21).

It follows from the above that the presence of spirality in this case has a weak effect on the cascade transfer of energy, because the relative value of spirality determined by the dimensionless ratio $F(k)/2kE(k)$ decreases monotonically with increasing k_0 independently of the spirality injected into the flow at wavenumbers $\sim k_0$. It is safe to assume that, in the case of sufficiently large k/k_0, the spirality should have only a minor effect on the dynamics and should be transferred and diffuse in the same way as a dynamically passive scalar admixture (Monin and Yaglom, 1996). At the same time, Kraichnan (1973) and Andre and Lesieur (1977) showed that, if the mode of generation of almost maximum spirality is realized for each wavenumber in the liquid flow under consideration, then the total transfer of kinetic energy toward higher wavenumbers should be weakened, and the process of the decay of turbulence should be extended in time. This leads us to an important conclusion that the relatively long existence of turbulence in a rotating protoplanetary cloud may be partially due to the lack of reflection symmetry with respect to the equatorial plane of the field of vortex velocities in the disk.

Hence, gyrotropic turbulence behaves in a way that is qualitatively different from the behavior of nonspiral turbulence. This allows us, in the case of possible realization of an inverse cascade of kinetic energy, not only to explain the development of the effect of negative viscosity in the differentially rotating (three-dimensional) protoplanetary cloud, but also to forecast the development of relatively stable and energy intensive coherent mesoscale vortex structures, which initiate the mechanisms of trigger cluster formation in the disk.

GENERATION OF SPIRALITY IN A ROTATING DISK

The lack of symmetry with respect to the $z = 0$ plane perpendicular to the vector of angular velocity Ω necessarily implies the breakdown of the mirror symmetry of random motions, which plays the crucial part in the generation of spirality (see Steenbeck et al., 1966). Let us show that such a symmetry is also absent in the case of a rotating protoplanetary disk stratified in z, where Archimedean forces ρ' g act on liquid elements whose density differs from the local ambient density ρ_0 by $\rho'\mathbf{g}$ (c $\mathbf{g} \cdot \Omega \ne 0$). In other words, let us analyze the role of the interaction of Archimedean and Coriolis forces in the generation of the mean spirality in the disk. Note in this connection that the lack of invariance with respect to the parity transform is a more general property than the presence of spirality, although it follows from the latter (see Gilbert et al., 1988).

Let us adopt the viewpoint that small-scale vorticity in the disk is produced by convection. To find the pseudoscalar function $\langle \mathbf{u} \cdot \text{rot}\, \mathbf{u}\rangle$, one has, in the general case, to solve the hydrodynamic problem in the Boussinesq approximation for the pulsating velocity field $\mathbf{u}$ in the presence of unstable density stratification and with a preferential direction of vortex twisting Ω (see, e.g., Eltayeb, 1972). We describe the motion of matter within the framework of this approach in a reference frame rotating at the average angular velocity Ω of the disk rotation. Equation of motion (1) then acquires the following form:

$$\partial_t\mathbf{U} + (\mathbf{U}\cdot\nabla)\mathbf{U} + 2\Omega\times\mathbf{U} = -\rho^{-1}\nabla P + \mathbf{g} + \nu\nabla^2\mathbf{U}, \tag{25}$$

where $2\Omega \times \mathbf{U}$ and $1/2|\Omega \times \mathbf{r}|^2$ are the Coriolis acceleration and the potential of the centrifugal force, respectively, and $\mathbf{g} = -\nabla(\psi - 1/2|\Omega \times \mathbf{r}|^2)$. The problem is characterized by two dimensionless numbers: the Rossby number $\mathbf{Ro} = U_0/\Omega L$ and the Ekman number $\mathbf{Ek} = \nu/\Omega L^2$. Here $L = O(R)$ is the scale length of the variation of the characteristic velocity U_0. The Rossby and the Ekman numbers for the disk are much less than unity; and therefore, for the sake of simplicity, we restrict our analysis to the geostrophic motion, when we can neglect the translational acceleration and the viscous term in formula (25). In this case, the equation for the turbulent component of the velocity field $\mathbf{U}$ in the Boussinesq approximation[39] takes the following form: $2\rho_0\Omega \times \mathbf{u} = -\nabla p' + \rho'\mathbf{g}$. We now apply the curl operation to this equation to obtain, in view of $\text{rot}\,\text{rot}(\rho'\mathbf{g}) = \nabla\rho' \times \mathbf{g}$,

$$2\rho_0\text{rot}(\Omega\times\mathbf{u}) = -\mathbf{g}\times\nabla\rho' \tag{26}$$

[39] Recall that in the Boussinesq approximation we can assume, up to the first order of small quantities and in view of relation $\nabla p_0 = \rho_0 g$, that $\rho^{-1}\nabla P + \mathbf{g} \approx \rho_0^{-1}(\nabla p' + \rho'\mathbf{g})$ with the simultaneous replacement of the continuity equation by the nondivergence condition, $\text{div}\,\mathbf{U} = 0$. Here, p' and ρ' are the pressure and density deviations from the main state P_0 and ρ_0 determined by the presence of winds and flows.

and, consequently,

$$
\begin{aligned}
2\rho_0(\Omega \times \mathbf{u}) \cdot \mathrm{rot}(\Omega \times \mathbf{u}) &= -(\Omega \times \mathbf{u}) \cdot (\mathbf{g} \times \nabla\rho') \\
&= -\Omega \cdot [\mathbf{u} \times (\mathbf{g} \times \nabla\rho')] \\
&= -(\Omega \cdot \mathbf{g})(\mathbf{u} \cdot \nabla\rho') + (\Omega \cdot \nabla\rho')(\mathbf{u} \cdot \mathbf{g}).
\end{aligned} \tag{27}
$$

We now set $\mathbf{u} = \mathbf{u}_\perp + \mathbf{u}_\parallel$ (where $\mathbf{u}_\parallel = u_z\mathbf{i}_z$ is the turbulent velocity component $\mathbf{u}$ that is parallel to the angular velocity vector Ω; $\mathbf{i}_z = \Omega/|\Omega|$; $u_z = \mathbf{u} \cdot \mathbf{i}_z$; $\mathbf{u}_\perp = u_x\mathbf{i}_x + u_y\mathbf{i}_y$ is the projection of velocity $\mathbf{u}$ onto the equatorial disk plane, which we hereafter assume to be uniform with respect to x and y; and $u_x = u_y = u_\perp$) and average formula (26) over horizontal planes z = const to obtain

$$
\begin{aligned}
2\rho_0|\Omega|^2\langle \mathbf{u}_\perp \cdot \mathrm{rot}\,\mathbf{u}_\perp\rangle &= -(\Omega \cdot \mathbf{g})\langle \mathbf{u} \cdot \nabla\rho'\rangle \\
&\quad + \mathbf{g} \cdot \langle \mathbf{u}\nabla\rho'\rangle \cdot \Omega.
\end{aligned} \tag{28}
$$

In the special case, where the vector $\mathbf{g}$ of gravity acceleration is parallel to the vector Ω of angular rotation and ρ_0 is uniform in the horizontal planes, it follows from formula (28) that, if the continuity equation $\nabla \cdot (\rho_0\mathbf{u}) = \rho_0\nabla_\perp\mathbf{u}_\perp + \partial(\rho_0 u_z)/\partial z = 0$ is applied, the following formula (Hide, 1976) is true:

$$
\langle \mathbf{u}_\perp \cdot \mathrm{rot}\,\mathbf{u}_\perp/2\rangle \cong -\frac{\langle \rho'\partial u_z/\partial z\rangle}{4\rho_0|\Omega|^2}(\Omega \cdot \mathbf{g}), \tag{29}
$$

which establishes the direct relation between the part $\langle \mathbf{u}_\perp \cdot \mathrm{rot}\,\mathbf{u}_\perp/2\rangle$ of the mean spirality H and the pseudoscalar $\Omega \cdot \mathbf{g}$. We now use the estimate (Moffatt, 1978)

$$
\begin{aligned}
\langle \mathbf{u} \cdot \mathrm{rot}\,\mathbf{u}\rangle &= \langle \mathbf{u}_\perp \cdot \mathrm{rot}\,\mathbf{u}_\perp\rangle + \langle \mathbf{u}_\perp \cdot \mathrm{rot}\,\mathbf{u}_\parallel\rangle \\
+ \langle \mathbf{u}_\parallel \cdot \mathrm{rot}\,\mathbf{u}_\perp\rangle &= \langle \mathbf{u}_\perp \cdot \mathrm{rot}\,\mathbf{u}_\perp\rangle\left(1 + \frac{\langle \mathbf{u}_\parallel \cdot \mathrm{rot}\,\mathbf{u}_\perp\rangle}{\langle \mathbf{u}_\perp \cdot \mathrm{rot}\,\mathbf{u}_\perp\rangle}\right) \\
&= \langle \mathbf{u}_\perp \cdot \mathrm{rot}\,\mathbf{u}_\perp\rangle\left(1 + O\left(\frac{U_\parallel/L_\perp}{U_\perp/L_\parallel}\right)\right) \\
&= \langle \mathbf{u}_\perp \cdot \mathrm{rot}\,\mathbf{u}_\perp\rangle(1 + O(L_\parallel^2/L_\perp^2)) \cong \langle \mathbf{u}_\perp \cdot \mathrm{rot}\,\mathbf{u}_\perp\rangle,
\end{aligned} \tag{30}
$$

and can assume that the spirality for the protoplanetary disk is approximately equal to $H \cong \langle \mathbf{u}_\perp \cdot \mathrm{rot}\,\mathbf{u}_\perp/2\rangle$, because the $L_\parallel/L_\perp$ ratio in this disk is sufficiently small. Here $U_\parallel$, $U_\perp$, $L_\parallel$, and $L_\perp$ are the characteristic velocities and spatial scale lengths parallel and perpendicular to the angular velocity vector Ω, which (by virtue of the continuity equation) are related by the formula $U_\parallel/L_\parallel = 0\,(U_\perp/L_\perp)$.

Note that the correlation $(g/\rho_0)\langle\rho' u_z\rangle$ also appears in the definition of the coefficients of turbulent transfer in the vertical direction and, in particular, in the definition of the coefficient of turbulent heat exchange in the vertical direction (see, e.g., Van Mieghem, 1973; Marov and Kolesnichenko, 2002), where it characterizes the rate of transformation of turbulent energy into the internal energy of the medium (or vice versa depending on the stability or instability of the distribution of density and temperature in the system) and thereby the degree of decay or generation of turbulence. Thus, relation (29) proves once again that nonzero spirality in the disk, which is generated by the interaction of Archimedean and Coriolis forces, indeed affects the time of turbulence sustenance in the disk.

Hence, the correlation between the density and the vertical velocity $\langle\rho'\partial u_z/\partial z\rangle$ plays an important role in the determination of the magnitude and sign of the mean spirality in the disk. The explicit form of this correlation and the form of the function H can be found only in terms of an adequate hydrodynamic model of the turbulent flow in the disk, where the spatial distributions of all hydrodynamic parameters are known. We do not do this here, but consider the qualitative pattern of the development of spirality in some convection zone of the disk. Let a vortex with the velocity u_z shifts by the distance ζ in the convection zone of the upper part of the disk ($0 < z < h_{\mathrm{disk}}$), whose mean density ρ_0 decreases from the equatorial plane toward the disk surface $[(-\partial\rho_0/\partial z) > 0]$. In this case, positive fluctuations of density, $\rho' \cong -\zeta\partial\rho_0/\partial z > 0$, must appear. Because of the gradient of the mean density, it must, in accordance with the continuity equation $\nabla_\perp\mathbf{u}_\perp \longrightarrow -\rho_0^{-1}\,\partial(u_z\rho_0)/\partial z$ (in the Boussinesq approximation), expand, i.e., acquire horizontal velocity components. The resulting momenta of Coriolis forces lead to the left-hand spiral rotation. The descending matter in the upper part of the disk should compress and rotate in the opposite direction under the action of the Coriolis forces; i.e., it must again undergo left-hand spiral motion.[40] Thus, convection in the upper half of the disk is more likely to lead to left-hand than to right-hand spiral motions. It is evident that right-hand spiral motions should dominate in the lower half of the disk ($-h_{\mathrm{disk}} < z < 0$). Hence, the lack of mirror symmetry with respect to the transform $z \longrightarrow -z$ leads to the enhancement of vortex structures of a certain sign. The balance of left- and right-hand helical motions may be established in the region of the equatorial plane of the disk only in the absence of the gradient of turbulence intensity in this plane (because uniform turbulence is conductive to the establishment of a uniform distribution of differently oriented vortices), because one of the types of spiral motions comes from below and the other type, from above, i.e., only at the latest stages of the evolution of a differentially rotating protoplanetary cloud.

[40] In the above reasoning, we neglected the intensity of turbulent pulsations in the convective part of the disk, which, in the general case, may not be everywhere the same. The anisotropic intensity of small-scale turbulence may, in principle, reverse the direction of rotation of an individual vortex in the convection zone. However, the density gradient is the main contributor to the total spirality H.

NEGATIVE VISCOSITY IN ROTATING DISK TURBULENCE AS A MANIFESTATION OF THE SPIRALITY CASCADE

Let us now proceed to the interpretation of the inverse cascade of the energy of pulsational velocity **u**, which is possible in the case of three-dimensional spiral turbulence, in terms of negative viscosity.

Problems of the momentum transfer theory. Let us begin with the fact that the authors of the vast majority of astrophysical papers use common hydrodynamic equations in their models of the evolution of a rotating turbulent cloud, but with turbulent viscosity instead of the molecular viscosity. In this case, the authors of the above-mentioned papers naturally use the linear relation

$$R_{ij} - \frac{1}{3}R_{kl}\delta_{kl}\delta_{ij} = \rho K_{ijmn}(\partial_m\langle U\rangle_n + \partial_n\langle U\rangle_m) = \rho K_{ijmn}e_{mn} \quad (31)$$

between the symmetric Reynolds stress tensor R_{ij} and the symmetric tensor $e_{mn} \equiv \partial_m\langle U\rangle_n + \partial_n\langle U\rangle_m$ of deformation rates (i.e., the general Prandtl theorem of the momentum transfer). The quantities K_{ijmn} (the components of some fourth-rank tensor that is symmetric in i, j and m, n) of this linear function have the meaning of the coefficients of turbulent viscosity and are determined by the statistical characteristics of small-scale turbulence. By the very definition of isotropic large-scale turbulence, all the mean quantities associated with it remain unchanged in the case of arbitrary rotations (but not necessarily in the case of reflections); the tensors that possess this property are isotropic.[41] Let us further assume that the tensor of turbulent viscosity, K_{ijmn}, is isotropic (but not mirror-symmetric). In this case, the expansion $K_{ijmn} = a\delta_{ij}\delta_{mn} + b\delta_{im}\delta_{jn} + c\delta_{in}\delta_{jm}$ is valid (see, e.g., (Korenev, 1996)) and we obtain, by substituting it into formula (31),

$$R_{ij} = -\frac{2}{3}\rho E\delta_{ij} + \rho\nu^{\mathrm{turb}}e_{mn} \ (\nu^{\mathrm{turb}} \equiv b + c). \quad (32)$$

Note that the coefficient of turbulent viscosity ν^{turb} in formula (32), which is determined by the small-scale field of pulsational velocity **u**, is usually assumed to be positive. However, we cannot rule out the exotic possibility $\nu^{\mathrm{turb}} < 0$,[42] which, according to Kraichnan (1976a), may be realized owing to the fluctuations of the spirality of the turbulent field **u**. We now use general formula (32) to obtain for the particular model of the averaged disk motion that the tangential stress does not depend on the gradient of angular velocity

$$R_{r\varphi} = \rho\nu^{\mathrm{turb}} r\frac{\partial}{\partial r}\left(\frac{\langle U\rangle_\varphi}{r}\right) = \rho\nu^{\mathrm{turb}} r\frac{\partial\Omega(r)}{\partial r}. \quad (33)$$

The angular velocity in the Keplerian disk decreases with heliocentric distance, and therefore the direction of transfer of angular momentum (of matter) should also be directed away from the Sun. Thus, if applied to the entire cloud,[43] the Prandtl theory of the angular momentum transfer (or the theory of turbulent stress) leads to a conclusion (an evidently wrong one) about the general outward transfer of matter throughout the entire rotating turbulent cloud.

Because of such problems faced by the theory of momentum transfer in the general case of curvilinear flows, the creators of semiempiric theory of turbulence Taylor (1915) and then Von Karman (1936) suggested a logical generalization of formula (33) where tangential stresses are assumed to depend on the gradient of momentum

$$R_{r\varphi} = \rho\nu_s^{\mathrm{turb}}\frac{1}{r}\frac{\partial}{\partial r}[r^2\Omega(r)]. \quad (34)$$

The difference between formulas (33) and (34) proved to be very important for the model of the evolution of the protoplanetary cloud, because angular velocity in the disk decreases with heliocentric distance, whereas the angular momentum increases, and hence the two quantities should be transferred in the opposite directions according to the above two viewpoints. That is why formulas (33) and (34), if used separately, cannot explain all the specific features of turbulent rotational motion of matter in all parts of the disk in the cases where outer parts of the cloud are effectively transferred outward and inner parts, toward the Sun (see Safronov, 1969). In view of this, Wasiutynski (1946) suggested a more general formula for tangential stresses $R_{r\varphi}$ in a rotating medium

$$R_{r\varphi} = \rho K_r^r\frac{1}{r}\frac{\partial}{\partial r}[r^2\Omega(r)] - 2\rho K_\varphi^\varphi\Omega(r), \quad (35)$$

which includes both cases considered above and is associated with the effect of anisotropic viscosity. In the case of a purely radial flow ($K_\varphi^\varphi = 0$), this formula transforms into formula (34), whereas in the case of an isotropic medium ($K_\varphi^\varphi = K_r^r$) it coincides with the common hydrodynamic formula (33). Note, however, that the form of the Wasiutynski stress tensor[44] (of which

[41]The Kronecker (δ_{ij}) and Levi-Civita (ε_{ijk}) tensors are examples of isotropic tensors of the second and third rank, respectively, and the tensor product $\delta_{ij}\delta_{mn}$ is a fourth-rank isotropic tensor. These tensors have the same components in all coordinate systems and hence have the invariable components in the case of arbitrary rotation.

[42]Note that the positiveness of the coefficients of turbulent exchange (and, in particular, of the coefficient ν^{turb}) has not been-proven in the general case for a turbulent liquid flow (unlike the case of molecular transfer coefficients, whose positiveness is deeply rooted in the thermodynamics of irreversible processes); turbulent viscosity can be negative in the case of a two-dimensional flow (see, e.g., Vergassola et al., 1993; Gama et al., 1994).

[43]Without the allowance for the effect of the strong gravitational field of the Sun on the inner parts of the protoplanetary cloud.

[44]Note that formula (35) is not a component of any tensor; it can be used only in a particular coordinate system.

formula (35) is a special case), which is widely used in the astrophysical literature (see, e.g., Tassoul, 1979) to explain the differential rotation of various cosmic objects in terms of "anisotropic viscosity," so far lacks physical argumentation; i.e., it is still unclear whether it is only a formal generalization or whether it characterizes turbulent flow more precisely. Below, we give a possible substantiation of formula (35) in terms of asymmetric hydrodynamics of turbulized media. However, we shall first show that negative viscosity may appear in three-dimensional disk turbulence even within the framework of the classical Prandtl theory of turbulent stress (i.e., formula (33)).

Negative viscosity (thermodynamical approach). In the case of phenomenological description of a quasi-equilibrium subsystem of structurized turbulent chaos, we proceed from the formalism of generalized statistical thermodynamics, which assumes the analysis of the ensemble of microscopically identical systems of turbulent chaos with identical generalized thermodynamical state parameters (such as the internal energy of chaos $U_{\mathrm{turb}}(\mathbf{r}, t)$ [45] the generalized "turbulization temperature" $T_{\mathrm{turb}}(\mathbf{r}, t)$[46], the specific volume $1/\rho(\mathbf{r}, t)$, etc.) and requires the use of a probabilistic approach (Kolesnichenko, 2002; Marov and Kolesnichenko, 2006). The latter is due to large-scale turbulent fluctuations of some additional parameters of the state of chaos,[47] $q_k(\mathbf{r}, t)$ ($k = \overline{1, n}$) (the so-called internal coordinates) which serve as a difference measure in any set of such thermodynamically identical systems. The internal coordinates, which describe the thermodynamic state of chaos, may include fluctuating positive-determined parameters, which adequately characterize a whirling liquid (including coherent mesoscale formations) inside a physically infinitesimal elementary volume $d\mathbf{r}$. In particular, one can choose the following stochastic quantities q_k [48] the turbulent energy dissipation rate ε, the generalized angular velocities (which characterize coherent mesoscale vortex formations), the enstrophy Ω (in the case of a flat flow), etc.

Kolesnichenko (2002) used methods of nonequilibrium thermodynamics to show that, in the case of our two-level macroscopic description of the turbulized medium of the protoplanetary cloud in terms of two interacting continua, in the vortex continuum, which corresponds to small-scale components of pulsating thermohydrodynamic parameters, such a quasi-stationary regime is established between the takeoff of energy from the "external source" (which, in particular, is associated with the averaged differential rotation of the turbulized matter of the disk) and the energy dissipation due to internal dissipative processes in the subsystem of chaos, where the production of entropy of turbulization $S_{\mathrm{turb}}(\mathbf{r}, t)$ is compensated by its outflow to the subsystem of averaged motion so that the total production of entropy from chaos is minimal [49] It follows from this that the subsystem of turbulent chaos exports entropy into the "external medium", i.e., gives it to the subsystem of averaged motion. In other words, the inflow of negative entropy (negentropy) from the "external medium" (the subsystem of averaged motion) is required to maintain the stationary state inside the subsystem of turbulent chaos; this negentropy coming to the subsystem of chaos is used for the maintenance and improvement of its internal structure. Such a condition is known (Prigogine and Stengers, 1984) to be sufficient for the development of coherent dissipative (mesoscale) structures in the vortex continuum.

Kolesnichenko (2002) showed that in this quasi-stationary case the total production of turbulent entropy (energy scattering) has the structure of a bilinear form $T_{\mathrm{turb}}\sigma_{\mathrm{turb}}(\mathbf{r}, t) = \sum_\alpha \Im_\alpha(\mathbf{r}, t) X_\alpha(\mathbf{r}, t)$, whose explicit form is determined by the particular model of turbulized medium, i.e., by the set of hydrodynamic processes included into the model. According to the main postulate of nonequilibrium thermodynamics (see, e.g., de Groot and Mazur, 1962), this form can be used to derive the defining (closing) relationships between the thermodynamic flows $\Im_\alpha$ and the forces X_α a in the form of linear relations $\Im_{\alpha i} = \sum_\beta L^{ij}_{\alpha\beta} X_{\beta j}$ ($\alpha, \beta = 1, 2, \ldots$). The peculiarity of the turbulized continuum is that the matrix of Onsager coefficients $L_{\alpha\beta}$ depends not only on averaged thermodynamical parameters of state of the medium (as in the laminar case), but also on the statistical characteristics of the subsystem of turbulent chaos and, in particular, on the energy flow ε along the cascade of turbulent vortices (which is thus one of the thermodynamic flows in the system) or on the flow of hydrodynamic spirality, which is effectively generated in the case of gyrotropic small-scale turbulence. Such a situation, which is typical of any self-organizing (synergetic) system, results, in the general case, in nonpositive-determined individual terms $\Im_\alpha(\mathbf{r}, t) X_\alpha(\mathbf{r}, t)$ in the sum $T_{\mathrm{turb}}\sigma_{\mathrm{turb}}(\mathbf{r}, t)$ which, however, is positive-deter-

[45] Turbulent chaos is much away from the full chaos of thermodynamic equilibrium because it possesses a certain orderliness: even in a developed locally isotropic turbulence, the Kolmogorov spectrum of the distribution of kinetic energy (of pulsational motion) in the space of wavenumbers k in the inertial scale interval $E(k) \sim k^{-5/3}$ is far from uniform ($E(K) = \mathrm{const}$).

[46] Note that the generalized temperature of the subsystem of turbulent chaos is not reduced in the general case to the absolute temperature.

[47] Large-scale turbulent fluctuations should be distinguished from statistical molecular fluctuations, arising due to the atomic structure of the system.

[48] Note that some of the internal coordinates q_k may belong to the incoherent component of the subsystem of turbulent chaos, whereas other internal coordinates may characterize individual coherent mesoscale structures.

[49] According to Kolmogorov (1941), the energy flow η along the hierarchy of turbulent vortices down to the molecular level is a characteristic parameter of the subsystem of small-scale turbulence. In the stationary case, this flow coincides with the energy dissipation rate ε.

mined, $\sigma_{turb} \geq 0$. It is well known (see, e.g., Haken, 1983) that in this case the superposition of various thermodynamic flows may in principle result in negative values of individual diagonal elements of matrix $L_{\alpha\beta}$ and, thereby, in a situation where some coefficients of turbulent exchange are negative.

Thus, in the case of the evolution of a turbulized protoplanetary cloud, we cannot exclude the possibility of the development of the situation when, in some parts of the cloud, modes of turbulent mass motion occur at which the coefficients of turbulent exchange may acquire negative values (e.g., the viscosity coefficient ν^{turb} in formula (33)) (see, e.g., Sivashinsky and Frenkel, 1992; Vergassola et al., 1993; Gama et al., 1994). It follows from the above analysis of gyrotropic disk turbulence that the hydrodynamic spirality H may serve as a statistical characteristic of the turbulent field, which could provide the inverse cascade of kinetic energy and thereby the appearance of the effect of negative viscosity in a three-dimensional disk.

Rotational viscosity. Let us now return to the difficulty of the Prandtl theory of momentum transfer in turbulized media that astrophysicists faced when they tried to explain differential rotations of gaseous astrophysical objects. The standard astrophysical approach to the derivation of the averaged hydrodynamic equations (based on the Reynolds postulates), which, in particular, are intended for modeling the protoplanetary cloud, cannot apparently be considered quite adequate, because, as we mentioned above, the actual picture of turbulent transfer in the disk essentially differs from the classical picture (see, e.g., Safronov, 1969). Although the authors of published works, beginning from O. Reynolds, the founder of the phenomenological theory of turbulence, and then G.D.Mattioli,[50] discussed the approaches involving the asymmetry of turbulent stress tensor ($R_{ij} \neq R_{ji}$) and such additional internal characteristics of the state of the turbulent field as the vortex, the moment of inertia, and the moment of internal forces, this line of research has been, unfortunately, neither deservedly appreciated nor further developed.

At the same time, recently interest has been renewed again in asymmetric hydrodynamics (hydrodynamics of moments)[51] of turbulized media due to certain achievements in the problem of spatial averaging of various equations of motion in continuum mechanics including, e.g., liquid flows in porous media, suspension-carrying flows, deformation of composite materials, etc. It was shown, in particular, (see, e.g., Ferrari, 1972; Nikolaevskiy, 2003), that more accurate spatial averaging (without the traditional Reynolds postulate of the commutativity of averaging and differentiating operations) of hydrodynamic equations for small elements of the continuum, made in order to describe the same motions on a macroscale, yield the generalized Reynolds equations. These averaged equations include, in particular, a term with rotational viscosity, which is associated with an asymmetric part of the turbulent stress tensor.

Nikolaevskiy (2003) used methods of hydromechanics of moments to derive for an asymmetric turbulent flow the following formula for energy dissipation associated with viscous processes:

$$T_{\mathrm{turb}}\sigma'_{\mathrm{turb}} = \left(\mathbf{R}^{\mathrm{s}} + \frac{2}{3}\rho E\mathbf{I}\right) : \mathbf{e} - \mathbf{R}^{\mathrm{a}} \cdot \boldsymbol{\varpi} + \mathbf{T} : \nabla(\mathrm{rot}\langle\mathbf{U}\rangle + \boldsymbol{\varpi}) \geq 0, \tag{36}$$

where $\mathbf{I}$ is the unit tensor; $\mathbf{e} = 1/2(\nabla\langle\mathbf{U}\rangle + \nabla\langle\mathbf{U}\rangle^{\mathrm{transp}})$ is the tensor of averaged deformations; $\mathbf{R}^{\mathrm{s}}$ and $\mathbf{R}^{\mathrm{a}}$ are the symmetric and asymmetric parts of the Reynolds stress tensor, respectively; $\boldsymbol{\varpi}$ ($\equiv\langle\mathrm{rot}\,\mathbf{u}\rangle$) is the so-called vector of internal angular velocity, which is due to the intrinsic vorticity of the field of pulsation velocities $\mathbf{u}$ and characterizes the vortex "anisotropy" of the flow on the microscale l[52]; $\mathbf{T}$ is the turbulent momentum stress tensor associated with pulsational transfer of fluctuations of the moment of momenta of small-scale vortices.[53] In the general case of an anisotropic liquid, flows and thermodynamic forces appearing in formula (36) are related by the following simple set of defining relationships:

$$R^{\mathrm{s}}_{ij} \equiv \frac{1}{2}(R_{ij} + R_{ji}) = -\frac{2}{3}\rho E\delta_{ij} + \rho K_{ijmn}e_{mn}, \tag{37}$$

$$R^{\mathrm{a}}_{ij} \equiv \frac{1}{2}(R_{ij} - R_{ji}) = -\rho K^{*}_{ijmn}\varepsilon_{mnk}\varpi_k, \tag{38}$$

$$T_{ij} = \rho K^{**}_{ijmn}\partial_n(\varepsilon_{mlk}\partial_l\langle U_k\rangle + \varpi_m), \tag{39}$$

typical of Kosser's asymmetric hydrodynamics. Here, the phenomenological turbulent coefficients K_{ijmn}, K^{*}_{ijmn}, and K^{**}_{ijmn} are strongly varying functions of averaged parameters of state of the medium and depend on the statistical characteristics of the turbulent velocity field $\mathbf{u}$.

In this paper, we consider only the simplest conclusions that follow from the isotropic (but not mirror symmetric) structure of turbulent transfer coeffi-

[50]See Mattioli's works published in 1933.

[51]Note that the asymmetric hydromechanics of Kosser (see, e.g., de Groot and Mazur, 1962) has since long gained wide recognition, e.g., in the theory of liquid crystals and liquid helium.

[52]Recall that, in the case of a disk, an elementary volume of the scale length l may contain a large number of rotating vortex formations (clusters). This, by the way, is a serious argument for the use of turbulent hydromechanics of moments and of the concept of two-level macroscopic description of turbulized medium in the models of a protoplanetary cloud.

[53]In asymmetric turbulent hydromechanics, this tensor appears in the additional evolutionary equation of the internal angular momentum balance (in the equation for $\boldsymbol{\varpi}$) (see de Groot and Mazur (1962); Nikolaevskiy (2003)).

cients[54]. Note that in most of the liquid, after a short relaxation time the curl of the averaged velocity, rot $\langle\mathbf{U}\rangle$, becomes equal to the angular velocity ϖ that determines the internal rotation of the mass elements of the continuum, $-\varepsilon_{mlk}\partial_l\langle U_k\rangle = \varpi_m$. In this case, the thermodynamic force in linear constitutive relations (39) also vanishes and so do the corresponding flux densities of the moments of momentum of small-scale vortices, i.e., the interaction between the vortices of the macroscopic velocity field and intrinsic rotational motion of the particles disappears (see, e.g., de Groot and Mazur, 1962). However, the law of the parity of tangential stresses $R_{ij} = R_{ji}$ on a macrolevel breaks down in the case of the turbulized continuum (unlike the behavior of the laminar flow) and relation (32) should be replaced by the following formula:

$$R_{ik} = R^{s}_{ik} + R^{a}_{ik} = -\frac{2}{3}\rho E\delta_{ik} + \rho\nu^{\text{turb}}e_{ik} + \rho\nu_r^{\text{turb}}\varepsilon_{ikp}(\text{rot}\langle\mathbf{U}\rangle)_p. \tag{40}$$

The coefficients ν^{turb} and ν_r^{turb} are determined by the field of turbulent velocity $\mathbf{u}$, and the coefficient ν^{turb} is a scalar, whereas the coefficient ν_r^{turb} is a pseudoscalar, because the tensor $\varepsilon_{ikp}(\text{rot}\ \langle\mathbf{U}\rangle)_p$ is a second-rank pseudotensor.

Let us now analyze turbulence with mirror symmetry. In this case, on the one hand, the coefficients ν^{turb} and ν_r^{turb} should not change during reflection transform; however, on the other hand, the coefficient ν_r^{turb} must reverse its sign, because it is a pseudoscalar. Therefore, the coefficient $\nu_r^{\text{turb}} = 0$ in the case of an isotropic and mirror symmetric small-scale turbulence. Hence, the rotational viscosity ν_r^{turb} may differ from zero only if the field of turbulent velocities is not statistically invariant with respect to the parity transform, in particular, if spirality $H \neq 0$.

In the disk turbulence model under consideration, relation (40) for shear stress acquires the following simple form:

$$R_{r\varphi} = \rho\nu^{\text{turb}}r\frac{\partial\Omega(r)}{\partial r} + \rho\nu_r^{\text{turb}}\frac{1}{r}\frac{\partial r^2\Omega}{\partial r}$$

$$= \rho\nu^{\text{turb}}\left(\frac{1}{r}\frac{\partial r^2\Omega}{\partial r} + 2\Omega\right) + \rho\nu_r^{\text{turb}}\frac{1}{r}\frac{\partial r^2\Omega}{\partial r} \tag{41}$$

$$= \rho(\nu^{\text{turb}} + \nu_r^{\text{turb}})\frac{1}{r}\frac{\partial r^2\Omega}{\partial r} - 2\rho\nu^{\text{turb}}\Omega$$

(compare it with formula (32)). Hence, the formula for tangential stresses in a rotating medium suggested by Wasiutynski can be physically substantiated in terms of asymmetric mechanics of turbulized media with an asymmetric Reynolds stress tensor. One of the important implications of formula (41) is the conclusion about the mutual complementarity of the Prandtl theory of momentum transfer and the Taylor vortex transfer theory in a rotating medium. The appearance of the additional degree of freedom ϖ in asymmetric hydrodynamics makes both approaches necessary for solving particular problems. In particular, one of the competing mechanisms should dominate in a rotating protoplanetary cloud depending on the numerical values of the coefficients of shear and rotational viscosity in the corresponding disk regions.

CONCLUSIONS

In conclusion, we briefly summarize the main results of our study. We analyzed the possibility of the influence of hydrodynamical spirality that develops in a rotating disk on the synergetic structurization of cosmic matter and on the appearance of the effect of negative turbulent viscosity in this disk within the framework of the problem of reconstruction of the evolution of the protoplanetary cloud surrounding the Sun at the early stage of its evolution. We showed that relatively long persistence of turbulence in a protoplanetary cloud may be partially due to the lack of mirror symmetry of the field of pulsating velocities with respect to the equatorial disk plane.

A specific mechanism of the development of coherent mesoscale formations in the subsystem of turbulent chaos associated with the phenomenon of phase-frequency synchronization of autooscillations of stochastic internal coordinates (corresponding to the coherent component of chaos) was analyzed in our earlier paper (Kolesnichenko, 2004). We also demonstrated (Kolesnichenko, 2005) the principal possibility of the self-organization of a flow, where the generation of coherent formations associated with the effect of "phase transitions" induced by the natural noise of the fine-grained fluctuating field of chaos is likely in the process of the temporal evolution of the quasi-equilibrium vortex subsystem. In this paper, we formulate the general concept of energy feeding of coherent mesoscale vortex structures in the thermodynamically open subsystem of turbulent chaos, which is associated with the realization of inverse cascade of kinetic energy in mirror asymmetric disk turbulence.

Because of the energy release, the inverse cascade generates the corresponding hierarchical system of gas condensations (with a fractal density distribution), which ultimately leads to the intensification of mechanical and physicochemical interactions between the par-

[54] In the case of an anisotropic small-scale turbulent field, the situation becomes much more complicated and a number of additional terms appear in formula (40), which are associated with the vector field (e.g., the field of the gradient of turbulence intensity, the field of density gradient, or the angular velocity of the system) that caused the anisotropy (see, e.g., Krause and Radler, 1980).

ticles of matter, resulting in spontaneous formation and growth of dust clusters, stimulation of condensation and phase transitions, of mass and heat exchange between different parts of the heterogeneous disk subsystem, and substantial modification of the oscillation spectrum, etc. In this case, gravity plays the crucial part at the final stage of the formation of large-scale gas and dust condensations in the domain of inner planets.[55]

We used the methods of nonequilibrium thermodynamics and a two-level description of maximally developed disk turbulence to demonstrate the possible appearance of the effect of negative viscosity in the three-dimensional case. We suggested that hydrodynamic spirality, which develops owing to fast rotation of an unstably stratified disk medium, can be used as a statistical characteristic of isotropic, mirror-noninvariant small-scale turbulence. In this case, the appearance of negative viscosity in the disk can be due to cascade transfer of energy from small to larger vortices in spiral turbulence.

We derived, within the framework of asymmetric hydromechanics of turbulized media, a formula for the tensor of turbulent stress in the Wasiutynski form, which is widely used in astrophysical literature to explain the differential rotation of various cosmic objects in terms of "anisotropic viscosity." This phenomenological relation, which is used in astrophysics because of the well-known difficulties faced by the Prandtl theory of momentum transfer in a rotating turbulized medium, has until recently lacked physical substantiation. In this paper, we reveal the mutual complementarity of the Prandtl theory of momentum transfer and the Taylor theory of angular momentum transfer for a rotating protoplanetary cloud in the cases where one of these competing mechanisms dominates (depending on the numerical values of the shear and rotational viscosity coefficients) at a certain heliocentric distance.

The ultimate aim of the approach that we elaborate in this paper is to develop a macroscopic model of the turbulent motion of liquid, which should be maximally close to reality and meet various dynamic conditions in natural media and in the protoplanetary cloud in particular. Many difficulties still remain to be overcome on this way, because the construction of a universal model of turbulence appears problematic. Our interest in the hydrodynamic spirality as applied to a structured disk turbulence stems from the fact that the existence of such an additional nonviscous invariant implies, generally speaking, a certain modification of the classical energy cascade process in the inertial spectral domain, where inverse energy transfer is possible from small to larger vortices. This makes it possible not only to explain the phenomenon of negative viscosity in a differentially rotating protoplanetary cloud, but also to forecast the birth of energy active coherent vortex structures, which ultimately trigger the formation mechanisms of gas and dust clusters in the disk. Unfortunately, the effect of inverse cascade of energy for three-dimensional gyrotropic turbulence so far lacks a reliable published confirmation in numerical simulations. Hence, we apparently have several more years to wait for an unequivocal and conclusive answer to the key problem of our approach to the criteria of realization of such a cascade in a differentially rotating disk.

[55]It is not improbable that there was no gravitational instability in the subdisk in the domain of inner planets (see, e.g., Safronov (1960)).

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